Modeling the dynamic effects of work experience on intrahousehold resource allocation

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Abstract

This paper analyzes intrahousehold allocation decisions in a repeated dynamic game between two married individuals. Two extreme cases are considered: collective maximization of the family welfare and a complete failure of cooperation within the family. In this simple intrahousehold allocation model each family member may choose how much time to work and how much time to spend in the household activities. Work in the market increases individual productivity through "learning by doing", while work at home results in the production of household public good. A genetic algorithm is used as a search method for optimal solution paths in both the cooperative and uncooperative game settings. Non-cooperative behaviors results in inefficiently low provision of household public good and failure of specialization. Potential policy interventions to reduce welfare loss in the case of non-cooperation are discussed.

1. Introduction

In this paper we model intrahousehold allocation choices as a repeated dynamic game between two married individuals. We believe that one of the many reasons why two individuals may choose to sign a marriage contract is the gains through specialization, so that each person could do what he or she is best at. Moreover, if people can gain comparative advantages through skill accumulation or because of differential investments in human capital, even on the onset identical individuals will (maybe randomly) specialize and enjoy the gains from increased productivities (see Becker, 1991 for extensive analysis).

We use a simple model of family decisions, where each family member may freely choose how to divide time between work for wages and work in the household. When working at home a family member produces household good which can be enjoyed by

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both spouses (it is essentially a public good). We introduce dynamics to the model by assuming, that individuals gain market productivity through work experience (for example, because of on the job training or simple learning by doing). Therefore, efficient family (where both family members cooperate to maximize family's welfare) should allow one spouse to specialize at the market work, while the other spouse would spend more time doing the housework, so that family could enjoy gains of experience through higher wages of employed spouse.

However, it is not so uncommon to see families where cooperation in the family seems to be absent. Reaching the cooperative agreement between spouses might involve significant time and energy costs. As a consequence, sometimes cooperation might fail and we will observe inefficient outcomes. We model such uncooperative outcome as a Nash equilibrium, where each spouse maximizes his or her own utility taking the other spouse's actions as given. The uncooperative equilibrium will in general result in the inefficiently low amount of public good provisions because of the free rider problem. Using computational methods, we analyze how this welfare loss depends on the model parameters. Then we offer potential policy tools to diminish the efficiency losses of such possible failure in cooperation.

The paper is organized as follows. Section 2 sets up the model and offers the analytical solution for a simple one period case. Both cooperative and non-cooperative equilibrium outcomes as well as implications of model parameter changes are considered. In section 3 we discuss the parameter choices and functional specifications of utility, production and wage determination functions. Section 4 presents numerical estimation of the model and interpretation of the results. Finally, section 5 concludes and offers extensions of the model.

2. The model

Following Ermisch (2003), we consider a simple intrahousehold allocation model with home production technology, where two family members, husband and wife, decide how much time to contribute to home production (housework, childcare, etc.) and how much time to work for wages (we abstract from leisure decisions). Spouses derive utility from consuming private good x_i bought in the market and public good G produced at home:

$$U^i = U^i\left(x_i, G\right),\ i = 1,2 \tag{2.1}$$

Production of the public good G requires only time inputs h_i of both family members in the following way:

$$G = G\left(h_1, h_2\right) \tag{2.2}$$

Let's normalize total time available to each spouse to 1. Assume spouses receive exogenous income y_i and wages w_i for time spent working. Then budget constraint (BC) for private good consumption (which is assumed to be perishable, i.e. it has to be consumed within the same period) is given by:

$$x_i = y_i + \left(1 - h_i\right) w_i \tag{2.3}$$

We abstract in the model from the process of family formation and we assume that divorce is not allowed or is excessively costly. Assume that spouses live together for a finite horizon T, that they discount their future consumption by β_i and that they can not save or borrow. If spouses can not reach the agreement within the family then they engage in the non-cooperative behavior. Then they maximize their utility functions (which are assumed to be separable over time) by choosing the amount of time contributions towards public good production while treating other spouse time input choices as given:

$$\max_{h_{it}} \sum_{t=1}^{T} \beta_i^{t-1} U^i\left(y_{it} + \left(1 - h_{it}\right) w_{it}, G\left(h_{it}, h_{jt}^*\right)\right);\ i, j = 1,2;\ i \neq j \tag{2.4}$$

The optimal solution to this problem is the Nash Equilibrium (NE) choice pairs $\left\{h_{1t}^*; h_{2t}^*\right\}_{t=1}^{T}$, which are self-enforceable, that is nobody would like to deviate, given that spouses do not have a possibility to re-optimize their behavior once the family game has started (this equilibrium is thus not Subgame Perfect (SPNE)).

Since family can be arguably viewed as a long-term (stationary) relation with repeated interactions, cooperative solution, which maximizes a weighted sum of both spouses' utilities subject to a pooled income constraint, is a more realistic outcome of the family game:

$$\begin{aligned} \max_{h_{1t},x_{1t},h_{2t},x_{2t}} \mu\left[\sum_{t=1}^{T}\beta_1^{t-1}U^1\left(x_{1t},G\left(h_{1t},h_{2t}\right)\right)\right]+ \\ +\left(1-\mu\right)\left[\sum_{t=1}^{T}\beta_2^{t-1}U^2\left(x_{2t},G\left(h_{1t},h_{2t}\right)\right)\right] \end{aligned} \tag{2.5}$$

$$s.t. \quad x_{1t}+x_{2t}=y_{1t}+y_{2t}+\left(1-h_{1t}\right)w_{1t}+\left(1-h_{2t}\right)w_{2t} \tag{2.6}$$

Here weight μ measures the bargaining power of the first spouse (say husband) and μ depends on the so called "treat points", or the best available alternative when cooperation breaks down. In our setup, the second best alternative is the non-cooperative equilibrium.

We assume that in competitive labor market individuals are paid wages equal to their productivity, and we hypothesize, that person's productivity depends positively on his or her accumulated work experience (possibly with decreasing returns) and negatively on time (skills depreciate due to changing technology or depreciating human capital). Moreover, we can also argue that previous period work hours have additional positive influence, because a person may loose technical skills if he is unemployed for some time. Then, wage equation takes the following form:

$$w_{it}=w_t^i\left(L_{it-1},\left(1-h_{it-1}\right),t\right);\ L_{it}=\sum_{k=1}^{t}\left(1-h_{ik}\right);\ i=1,2 \tag{2.7}$$

Analytic solution of Static Cobb-Douglas – linear case

If we assume that Utility function is Cobb-Douglass, $U(x_i, G) = x_i^{\alpha} G^{1-\alpha}$, where α is the same for both spouses, while public good production technology is linear in family members time inputs, $G = P\left(\kappa h_1 + (1-\kappa) h_2\right)$, then we can solve the above model analytically in a static (one period) case.

Cooperative equilibrium

If reaching the agreement between spouses is not too costly, family members can potentially achieve the cooperative equilibrium in their intrahousehold bargaining game. By assumption, this “collective” equilibrium is Pareto efficient, i.e. we can not improve one spouse’s situation without hurting the other spouse†. However, the actual outcome in this collective equilibrium depends on the bargaining power of each spouse which is captured by parameter μ. In what follows, we do not consider how the bargaining process takes place and therefore we take μ as given. Then cooperative solution can be found by maximizing family’s welfare function, which is formulated as a weighted sum of individual spouses’ utilities, subject to a pooled income budget constraint (BC):

$$\max_{h_{1t}, x_{1t}, h_{2t}, x_{2t}} \mu\left[x_1^{\alpha} G\left(h_{1t}, h_{2t}\right)^{1-\alpha}\right] + (1-\mu)\left[x_2^{\alpha} G\left(h_{1t}, h_{2t}\right)^{1-\alpha}\right] =$$

$$= P^{1-\alpha}\left[\mu x_1^{\alpha} + (1-\mu) x_2^{\alpha}\right]\left(\kappa h_1 + (1-\kappa) h_2\right)^{1-\alpha} \qquad (2.8)$$

$$s.t. \qquad x_1 + x_2 = y_1 + y_2 + (1-h_1) w_1 + (1-h_2) w_2; \; h_1, h_2 \in [0,1]$$

Denote $\tilde{\mu} = \dfrac{1-\mu}{\mu}$ and $\tilde{\kappa} = \dfrac{1-\kappa}{\kappa}$. Then this problem is equivalent to solving the following Lagrangian:

$$L = \left(x_1^{\alpha} + \tilde{\mu} x_2^{\alpha}\right)\tilde{G}^{1-\alpha} + \lambda\left[y - x_1 - x_2 - w_1 h_1 - w_2 h_2\right] \qquad (2.9)$$

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† Chiappori (1992) was among the first to use the fact that finding Pareto efficient intrahousehold allocations is equivalent to maximization of weighted sum of individual utilities, where $\tilde{\mu}$ can be interpreted as Lagrange multiplier associated with the Pareto efficiency constraint: max $U^1\left(x_1, G\right)$, subject to $U^2\left(x_2, G\right) \geq \overline{u}_2$.

where $y \equiv y_1 + y_2 + w_1 + w_2$ is the "full income" (the maximum income which could be earned if both spouses would work for wages full time) of the family and $\tilde{G} = G/(P\kappa) = h_1 + \tilde{\kappa} h_2$ is rescaled public good. Since $\tilde{G}(h_1, h_2)$ is a linear function in h_1 and h_2, we will have corner solutions, if $\tilde{\kappa} \neq \frac{w_2}{w_1}$. For example, if $\tilde{\kappa} > \frac{w_2}{w_1}$ then $h_1 = 0$ and $h_2 \leq 1$. If we consider the case where $\tilde{\kappa} = \frac{w_2}{w_1}$, then $w_1 h_1 + w_2 h_2 = w_1 \left(h_1 + \frac{w_2}{w_1} h_2 \right) = w_1 \left(h_1 + \tilde{\kappa} h_2 \right) = w_1 \tilde{G}$. Then BC becomes $x_1 + x_2 + w_1 \tilde{G} \leq y$ which implies that household members can freely choose who should work for wages and who should stay at home, since in this case relative productivities of spouses are equal. Solution of problem in (2.9) implies, that in cooperative equilibrium spouses will choose to produce:

$$G^{*CE} = (1-\alpha) yP \frac{\kappa}{w_1} = (1-\alpha) yP \frac{1-\kappa}{w_2} \tag{2.10}$$

(G^{*CE} will be equal to the bigger of these two expressions if $\tilde{\kappa} \neq w_2/w_1$). Amount of private consumption will be determined according to the bargaining power of family members, as measured by parameter $\mu' = \left(\frac{\mu}{1-\mu} \right)^{\frac{1}{1-\alpha}}$:

$$x_1^{*NE} = \frac{\mu'}{1+\mu'} \alpha y \quad \text{and} \quad x_2^{*NE} = \frac{1}{1+\mu'} \alpha y \tag{2.11}$$

Therefore, in cooperative equilibrium family members will behave as if they were allocating part of family's "full income", $(1-\alpha) y$, to pay for public good production, and then sharing the rest, αy, (according to their bargaining power) to pay for their private consumption goods. Reallocation of exogenous income from one spouse to another does not change the amount of total income, so it should not change the cooperative equilibrium amount of produced household public good.‡ However, in general, such

‡ A good example of such reallocation is the change in the assignment who receives child benefits (father or mother), which are exogenous given the quantity of children. See Lundberg at al. (1997) for an empirical example in the UK.

reallocation will change the bargaining power of the spouses, which will influence their equilibrium private consumption.

Non-cooperative equilibrium

If cooperation can not be achieved, family members play a non-cooperative game. Let's assume that spouse 1 (say husband) can also make unilateral transfer to his wife, s_1, but for the time being, assume that the amount of transfer is determined exogenously (for example, by social norms). We can substitute constraints for private and public goods into utility function and find the first order conditions with respect to time inputs, which gives us the reaction functions for spouses' time allocation:

$$\max_{h_i} U\left(x_i, G\right) = \left(\left(1-h_i\right)w_i + y_i - s_i\right)^{\alpha}\left(P\left(\kappa h_1 + \left(1-\kappa\right)h_2\right)\right)^{1-\alpha}, where\, s_2 = -s_1 \qquad (2.12)$$

$$\Rightarrow\; h_1^*\left(h_2\right) = \frac{\left(1-\alpha\right)\left(1\cdot w_1 + y_1 - s_1\right)}{w_1} - \alpha h_2\left(\frac{1-\kappa}{\kappa}\right) \qquad (2.13)$$

$$\Rightarrow\; h_2^*\left(h_1\right) = \frac{\left(1-\alpha\right)\left(1\cdot w_2 + y_2 + s_1\right)}{w_2} - \alpha h_1\left(\frac{\kappa}{1-\kappa}\right) \qquad (2.14)$$

Here $1\cdot w_i + y_i - s_i$ shows the maximum amount of income one could earn, working all the time for wages, which is usually referred to as full income. We can think of wage as a price on time devoted to home production (foregone wage earnings), while higher productivity at home makes private good relatively more costly. Equations (2.13) and (2.14) indicate that in non-cooperative case each family member's optimal time allocation towards the public good production increases with his/her productivity (substitution effect), increases with income (minus net subsidies) and decreases as the other spouse's allocated time increases (free-rider effect). Allocated time also increases with one's wage if exogenous income minus net subsidies is positive (income effect dominates), but it decreases with one's wage if income minus net subsidies is negative (substitution effect dominates) and the level of wages has no effect on allocated hours if $y_i - s_i$ is zero. From equation (2.12) we can clearly see that if husband is allowed to choose the amount of transfers, he will choose not to subsidize his wife ($s_1 = 0$), as his utility decreases with s_1.

Denote $A_i \equiv \frac{1 \cdot w_i + y_i - s_i}{w_i}$ (value of full income in work time units) and $B \equiv \frac{\kappa}{1-\kappa}$ (husband's and wife's productivities ratio) then

$$h_1^*\left(h_2\right) = \left(1-\alpha\right) A_1 - \frac{\alpha}{B} h_2 \text{ and } h_2^*\left(h_1\right) = \left(1-\alpha\right) A_2 - \alpha B h_1 \qquad (2.15)$$

which we can solve simultaneously to find Nash Equilibrium (NE) time allocation outcomes. Assuming internal solution, which we can get if husband's and wife's productivities are not too different and when income and wages of spouses are similar (i.e. when A_1 and A_2 are of similar magnitude and B is close to one)[§]:

$$h_1^{*NE} = \frac{A_1 - \left(\alpha/B\right) A_2}{1+\alpha} \quad \text{and} \quad h_2^{*NE} = \frac{A_2 - \left(\alpha B\right) A_1}{1+\alpha} \qquad (2.16)$$

Equilibrium amount of household public good when both spouses are contributing is given by:

$$G^{*NE} = P\left(\kappa h_1^* + \left(1-\kappa\right) h_2^*\right) = P \frac{\left(1-\alpha\right)\left(\frac{k}{w_1}\left(w_1 + y_1 - s_1\right) + \frac{(1-k)}{w_2}\left(w_2 + y_2 + s_1\right)\right)}{1+\alpha} \qquad (2.17)$$

When spouses' productivities weighted by market wages are equal: $\frac{\kappa}{w_1} = \frac{1-\kappa}{w_2}$, then $G^{*NE} = P\frac{\left(1-\alpha\right)}{1+\alpha}\frac{(1-k)}{w_2}\left(w_1 + y_1 + w_2 + y_2\right) = \frac{G^{*CE}}{1+\alpha}$. In this case we can easily see that public good will be under-provided in the non-cooperative equilibrium[**].

An interesting question from policy perspective is what could be done to improve the non-cooperative equilibrium outcome. The potential policy tools are differential taxation of wage income, which could equalize the effective after-tax wage rates of wife and husband or alternatively make them more diverse; and redistribution of exogenous income between spouses. Bergstrom et al. (1986) in their influential paper show that if

[§] When $A_1 = A_2 = 1\left(\text{or } y_i - s_i = 0\right)$ and $\alpha = 1/2$, then $h_i^{*NE} \in \left(0,1\right)$ if $2 > B > 1/2$ or $1/3 < \kappa < 2/3$. So for $A_1 = A_2 = 1$ and $\alpha = 1/2$, in non-cooperative equilibrium, husband will not contribute to home production at all if wife is at least twice as productive as him.

[**] Intuition suggests that non-cooperative equilibrium will result in under-provision of the public good for most parameter values, as spouses will tend to free-ride on each other.

public good is purchased (instead of production at home), then small redistribution of income (which does not change the set of contributors towards the public good) will not change the equilibrium amount of the public good.

We can see that in our case Bergstrom et al. neutrality result holds only if spouses' productivities weighted by market wages are equal: $\frac{\kappa}{w_1}=\frac{1-\kappa}{w_2}$. So, if husband earns higher wages or if he is somewhat less productive at home, $\frac{\kappa}{w_1}<\frac{1-\kappa}{w_2}$, then redistribution of income from husband to wife (which is equivalent to positive subsidy, $s_1>0$) will lead to the increase in non-cooperative equilibrium amount of public good produced and consumed in the household.

If in equilibrium only one household member contributes to home production (say wife), then $h_1^{*NE}=0$, $h_2^{*NE}=(1-\alpha)(w_2+y_2+s_1)/w_2$ and the amount public good production is given by equation (2.18). This will be equal to the cooperative equilibrium amount in equation (2.10) only if wife will be sufficiently subsidized (if $s_1=y-w_2+y_2=w_1+y_1$), i.e. if husband is forced to give away all of his income. We can be sure that in our society such policy will never be approved, and we again will have significant under-provision of the household public good:

$$G^{*NE}=P(1-\kappa)(1-\alpha)(w_2+y_2+s_1)/w_2 \tag{2.18}$$

Consider how spouses' utility change as κ increases, i.e. as husband becomes more productive relative to wife. Assume that $y_1=y_2$ and $s_1=0$, then $A_1=A_2$. If both spouses contribute to public good production, then $\frac{\partial G^{*NE}}{\partial\kappa}=0$, while $\frac{\partial x_1^{*NE}}{\partial\kappa}=-w_1A_2\frac{\alpha}{1-\alpha}\frac{1}{\kappa^2}<0$, and $\frac{\partial x_2^{*NE}}{\partial\kappa}=w_2A_1\frac{\alpha}{1-\alpha}\frac{1}{(1-\kappa)^2}>0$. So, a small increase in husband productivity at home would hurt his equilibrium utility as he would choose to work more at home and would earn less labor income (this of course would make wife happier).[††]

[††] Maybe this can explain why in some families husbands pretend to be lousy cookers and so ineffective in housekeeping.

3. Functional form specifications and parameter choices

A simple one period intrahousehold allocation model with Cobb-Douglas utility functions and a linear production function of the previous section gives powerful predictions, but it might be too restrictive in some settings. It is of interest to solve this model for multi-period case by allowing more flexible functional forms. This can not be solved analytically, but computational results can be readily obtained. To estimate the model numerically we need to assume specific functional forms and parameter values. We assume that utility function is CES, where α measures the relative preference towards private versus public good, while ρ measures the inequality between x_i and G aversion (or alternatively the degree of substitutability between these two goods – the smaller ρ, the lower the degree of substitutability):

$$U^i = \left(\alpha x_i^\rho + (1-\alpha) G^\rho\right)^{\frac{1}{\rho}};\ 0 < \alpha < 1;\ \rho \le 1;\ i = 1,2 \tag{3.1}$$

CES function is linear homogenous with elasticity of substitution parameter equal to $\sigma = 1/(1-\rho)$. When $\rho = 1$, utility function becomes linear, $U(x_i, G) = \alpha x_i + (1-\alpha) G$, when $\rho \to 0$, functional form of utility is Cobb-Douglass, $U(x_i, G) = x_i^\alpha G^{1-\alpha}$, and finally, when $\rho \to -\infty$, the indifference curves become L-shaped, i.e. we get Rawlsian utility function: $U(x_i, G) = \min(x_i, G)$. In general, we can allow parameters α and ρ to differ between spouses, but in my computations we assume that they are equal.

Similarly, we assume that home production function is CES, where κ measures husband's relative productivity (as compared to wife's) at home production, while λ measures the degree of substitutability of family members' time in home production, P is just a scale parameter, and θ is returns to scale parameter (we have constant returns to scale production when $\theta = 1$):

$$G = P\left(\kappa h_1^\lambda + (1-\kappa) h_2^\lambda\right)^{\frac{\theta}{\lambda}};\ 0 < \kappa < 1;\ \lambda \le 1;\ \theta, P > 0 \tag{3.2}$$

We could argue that for some household public goods spouses' time might be perfectly substitutable (for example, cooking), while other public goods like child quality might require time input of both spouses in production. So, we can vary λ to describe these different cases.

Finally, intrahousehold allocation pattern highly depends on individual wages. In competitive labor market individuals should be paid wages equal to their marginal productivity, where productive characteristics arguably depend on the amount of so called Human capital. When modeling labor earnings, researchers most commonly use Mincer's (1974) "human capital earnings function", where log wages are modeled as the sum of a linear function of years of education and a quadratic function of years of potential experience (age minus years of education minus six).

A better measure of work experience is the cumulated sum of work hours through the lifetime, since people who work full time should accumulate more skills than casual workers. If workers have capacity to learn by doing, hours worked for wages will serve not only as an income source but also as investment to human capital and productivity. We can capture this wage dynamics by assuming that wage function takes the usual log-linear form, where decreasing returns to experience are allowed by including the quadratic term. L_{it} denotes accumulated work experience, t is just a time trend, and $(1-h_{it-1})$ measures work hours in the previous period:

$$\log(w_{it}) = \gamma_0 + \gamma_1 L_{it-1} + \gamma_2 L_{it-1}^2 + \gamma_3 (1-h_{it-1}) + \gamma_4 t;\ L_{it} = \sum_{k=1}^{t} (1-h_{ik});\ i = 1,2 \qquad (3.3)$$

Using data for the years 1968-1997 fr0m the Panel Study of Income Dynamics (PSID) conducted by the University of Michigan (in the later years this survey was conducted only biannually), we estimate wage regression (3.3) coefficients in order to get reasonable time paths for wages. We define annual hours above or equal to 2500 as full employment ($(1-h_{it})=1$) and we use only observations for individuals with non-missing information about their work hours for all years after they finished their education (so for example, the person who got a college degree in 1968 was 50 years old in 1997 with at most of 29 years of accumulated work experience in that year). Fixed effects (which accounts for individual heterogeneity) and random effects estimation yield similar results which are summarized in table 1 on the next page. We should note, that after accounting for accumulated work hours, the usually used potential experience variable becomes insignificant, which confirms my skepticism about the usefulness of the latter measure.

Table 1. Estimated log-wage regression

Regressors:	White Males				White Females				Both sexes			
	FE		RE		FE		RE		FE		RE	
	coef	s.e.	coef	s.e.	coef	s.e.	coef	s.e.	coef	s.e.	coef	s.e.
schooling	-	-	0.0915	0.0053	-	-	0.1188	0.0045	-	-	0.1070	0.0035
L_{it-1}	0.0655	0.0046	0.0642	0.0026	0.0716	0.0043	0.0769	0.0032	**0.0624**	0.0026	0.0670	0.0019
L_{it-1}^2	-0.0014	0.0001	-0.0014	0.0001	-0.0019	0.0002	-0.0020	0.0002	**-0.0014**	0.0001	-0.0016	0.0001
$(1-h_{it-1})$	0.1680	0.0126	0.1630	0.0123	0.2869	0.0126	0.2954	0.0123	**0.2367**	0.0089	0.2374	0.0087
time	-0.0067	0.0030	-0.0045	0.0014	-0.0057	0.0015	-0.0070	0.0010	**-0.0038**	0.0012	-0.0050	0.0008
male	-	-	-	-	-	-	-	-	-	-	0.1728	0.0147
constant	2.2263	0.0297	0.9738	0.0769	1.9497	0.0170	0.3843	0.0625	**2.0476**	0.0132	0.5479	0.0490
individuals	1443				1915				3358			
observations	19041				23003				42044			

4. Numerical estimation results

We use Genetic Algorithm (GA) as a maximization procedure[‡‡]. The principles of GA maximization and its advantages are described in the Appendix B. We solve for noncooperative Nash equilibrium using two parallel GA players. In the process of optimal search GA players exchange their best chromosomes simultaneously, so that the best responses, given action of the rival player, could be determined. The chromosomes are exchanged using shared memory, which is implemented using Matlab *memmapfile* object constructor. We assume that spouses can not borrow or save and therefore budget constraint for private good has to be satisfied every period. We use this budget constraint (BC) to express the private good consumption in terms of time allocation choices, h_{it}, and then maximize each player's intertemporal utility over the space of $\{h_{it}\}_{t=1}^{T}$, where $h_{it} \in [0,1]$ for each player i.

[‡‡] Estimation is performed with Matlab v.7, using GOAT algorithm by C.R. Houck, J. Joines, and M. Kay, "A genetic algorithm for function optimization: A Matlab implementation". ACM Transactions on Mathematical Software, submitted 1996.

Cooperative solution requires only one artificial GA player which maximizes a weighted sum of both spouses' utilities subject to a pooled budget constraint. Since we have only one budget constraint, we use this BC to express x_{1t} in terms of x_{2t}, h_{1t}, and h_{2t}, and then maximization is performed over 3*T variables: $\{h_{1t}, h_{2t}, x_{2t}\}_{t=1}^{T}$. To ensure that optimal variable paths are within bounds we use a penalty function approach ($x_{it} \geq 0; 0 \leq h_{it} \leq 1; i = 1,2; \forall t$). Since individuals discount their future consumption, we estimate average flows of variables (as summary statistics) using the following formula: $\overline{x} = \frac{\beta^{T}-1}{\beta-1}\sum_{t=1}^{T}\beta^{t-1}x_t$, which is derived from relation: $\sum_{t=1}^{T}\beta^{t-1}x_t = \sum_{t=1}^{T}\beta^{t-1}\overline{x} = \overline{x}\frac{\beta^{T}-1}{\beta-1}$.

We set $T = 25$, which seems a long enough period to capture most interesting dynamics in wages and consumption choices. Wage equation parameters are assumed to be similar to the estimated coefficients from the PSID data and are summarized in table 2. As table 2 shows, at the beginning of their life time female workers are assumed to earn about 20% lower wages than males (which could be referred to as female discrimination), while returns to experience are assumed to be identical for both spouses. We assume that individuals discount their future by 5% subjective discount rate, and we assume that utility of spouses is weighted equally when game outcome is cooperative. In a typical GA optimization we set the size of population to 80 and we allow the algorithm to search for an optimum until the maximum GA generation (typically set to be equal 10000) is reached. The exogenous income is assumed to be constant in all periods and equal to 1 for both spouses.

Table 2. Model parameter values for case 1

	Husband	Wife		Husband	Wife		
Wage equations			**Utility functions**			**Production function**	
γ_0	2.2	2	β	0.9524	0.9524	P	10
γ_1	0.065	0.065	α	1	1	κ	0.5
γ_2	-0.0014	-0.0014	ρ	0	0	λ	1
γ_3	0.15	0.15	**Pareto weight in cooperative case**			θ	1
γ_4	-0.004	-0.004	μ	0.5			

Naturally, as our 1st case we consider Cobb-Douglas utility functions and a linear production function, for which the analytical solution was derived in section 2. To single out the impact of wage differences we assume that spouses are equally productive at home. The full set of assumed parameter values for case 1 is summarized in table 2. Matlab estimation results are presented in the Appendix A. As panel (a) of figure A1 shows, when spouses fail to achieve cooperation household public good will be underprovided, with both spouses spending most of the available time to work for wages (husbands in this case works less at home since he earns higher wages – substitution effect). The kinks in the first and the last periods of time paths is a result of the assumption that previous period work hours have a significant positive impact on wages – in the first period work experience is zero, while in the last period spouses work somewhat less, since they will not be able to utilize the benefits of increased work experience in the later periods. From the estimation output we can see that both spouses suffer utility loss when cooperation is failed, although the loss is lower for husband, who can compensate lower amount of public good by increasing his private consumption, as he earn higher wages. As we would expect the cooperative case results is full specialization, since this allows for the family to enjoy full returns on husband's work experience.

In the second and third cases we set $\rho = -5$ and $\lambda = -10$, which assumes that spouses' time inputs can not be easily substituted in home production and family members also can not simply replace public good with private food (child quality could be an example of a such household public good case). We want to consider the impact of a change in spouses' productivity at home on the equilibrium outcomes. Figure A2 in the Appendix illustrates the case when spouses are equally productive ($\kappa = 0.5$). The cooperative equilibrium is similar as in case 1, but now husband devotes some time to housework activities, especially as he gets older. In non-cooperative equilibrium the under-provision of the public good is less severe, since husband contributes a significantly higher amount of time to home production even if his wages are higher.

Consider an increase in wife's relative productivity at home production ($\kappa = 0.3$). The equilibrium outcomes of this change are depicted in Figure A3 in the Appendix. In both cooperative and uncooperative equilibriums husband's time inputs in home production decline and the degree of specialization in cooperative case increases. An interesting question is how spouses' utilities are affected by such change. Cooperative case is clearly better after such productivity decline, since joint utility increases due to higher specialization in the family. In uncooperative equilibrium husband becomes better off, while wife's utility declines. Because wife becomes more productive she spends more time at home production, which forces the selfish husband to decrease his time input, and which leaves the overall production of public good even lower (this is because we assumed that λ is highly negative).

In the final, 4th, case we consider the reallocation of exogenous income from husband to wife ($y_{1t} = 0$ and $y_{2t} = 2$) leaving all the other parameter values as in case 3. As we can see from Matlab output in the Appendix A, this small redistribution of income slightly raises the non-cooperative equilibrium amount of G, while cooperative equilibrium remains unchanged (since overall income of the family does not change). In uncooperative case wife spends more time working at home, while husband free-rides at the expense of wife's higher time input and he works longer hours for wages. Thus, husband can compensate the decrease in exogenous income by his higher labor income and enjoy more of both private and public good, while for some parameter values wife's private good consumption may even declines and she can become even worse after such reallocation (for example, in case 4, both spouses' utilities increase after this income reallocation).

5. Concluding remarks

This paper used genetic algorithms to solve for optimal intrahousehold allocation choices in the presence of public goods and analyze how welfare loss of uncooperative behavior depends on the model characteristics. Probably most of the marriages can be considered to be in a cooperative long term agreement. In this case our model captures the Becker's (1991) prediction that efficient family should specialize if productivity can be improved through investments in human capital, which in our case is a simple learning by doing.

However, if due to some reasons cooperation fails, one possible outcome of such failure is a simultaneous non-cooperative Nash equilibrium. Such uncooperative behavior in all considered cases results in inefficiently low provision of the public good (free-rider problem) and the failure of specialization. As a policy solution to uncooperative inefficiency we suggest reallocating money resources to a more productive spouse, which would increase both the amount of G provided and the degree of specialization. Alternatively we could tax wage income and subsidize the other spouse, which should have similar implication.

Collective equilibrium and completely uncooperative behavior are just two extreme cases. However our analysis can be readily extended to consider the intermediate cases. For example, even if spouses currently live in a tacit agreement, existence of a positive probability of failure in cooperation, will have similar implications as the uncooperative game itself: spouses may not fully specialize and contribute too little to home production, if they fear that uncooperative period may come. Another possible extension of the model is to allow for accumulation of skills in home production, which would bring in additional gains to specialization in the marriage. We can also easily introduce Becker's "altruism" in the family, i.e. we can allow each spouse's utility to enter directly into the other spouse's utility function. This would say that each family member cares about the other individual's wellbeing in the family and this should mitigate the free-rider problem in the non-cooperative equilibrium, while cooperative equilibrium would not change as spouses were already maximizing their joint family welfare.

An additional interesting extension would be Stackelberg leadership game as the non-cooperative outcome. Arguably, in some families one of the spouses could behave as a natural leader and move first (for example, by choosing a career job which requires long work hours), while the other spouse would make her or his decisions after observing the leader's choices. This type of the game should be solved backwards, i.e. the Leader would maximize his utility taking into account the reaction function of the follower. Alemdar and Sirakaya (2003) suggest approximating the follower's reaction function by Neural Network and then solving the game similarly as in this paper. However, in our case the problem is highly multidimensional and would require estimating at the minimum $T \cdot 2^T$ coefficients for the simplest linear approximation. Therefore, an offline estimation proposed by Vallee and Basar (1999) would be a preferable estimation method for our problem. Offline estimation requires the Follower to solve for the optimal response for each Leader's trial strategy, which might be a very slow process, but the best response optimization could be shared by a few (five or ten) parallel GA players (followers), which would expedite the process.

Finally, as we have already discussed, our equilibrium solutions are not Subgame Perfect (SPNE), because spouses could decide to re-optimize their behavior after the family game has started and this could lead to a different solution path. We could solve this game backwards (starting from the last period, where the wage would be a function of all previous time input choices) to ensure that our solution is time-consistent. Vedenov and Miranda (2001) offer a promising estimation method for a time-consistent solution of the dynamic game. Our family game can be formulated as a dynamic programming problem, where accumulated work experiences can be treated as state variables, while time contributions towards public good would be control variables. Then the dynamic game can be expressed as a set of two simultaneous Bellman equations and then solved iteratively for the infinite horizon problems, or backwards starting form the last period for the finite horizon problems. Infinite horizon setting can be attractive when modeling small daily intra-household allocation decisions, while finite horizon game is more appropriate for more important household problems like career choices, investments in children quality or housing.

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Appendix A

Matlab Estimation Output

Case 1

```
Started at 09-Jun-2005 23:57:10
***************************************************************************
********  Intrahousehold Allocation Model with Learning by Doing **********
**************************** Player 1 *************************************

Parameters for player    1
Time_periods            25
Population_size         80
Max_GA_generation      5000
Utility fn parameters:    beta       alpha      rho
                          0.95238         0.5           0
Production fn parameters: P          kappa       theta       lambda
                          10          0.5           1           1
Wage fn parameters:       gamma0    gamma1      gamma2      gamma3     gamma4
                          2.2       0.065     -0.0014        0.15      -0.004
Average value of private good - x:                     11.0063
Average value of wage - w:                             13.9939
Average value of exogenous income - y:                 1
Average value of public good - G:                      2.8993
Average value of time in public good production (h):   0.28034
Estimated average individual utility:                  5.6467

***************************************************************************
Finished at 10-Jun-2005 00:08:42
Total estimation time 691.765 seconds.

Started at 09-Jun-2005 23:57:12
***************************************************************************
********  Intrahousehold Allocation Model with Learning by Doing **********
**************************** Player 2 *************************************

Parameters for player    2
Time_periods            25
Population_size         80
Max_GA_generation      5000
Utility fn parameters:    beta      alpha      rho
                          0.95238         0.5           0
Production fn parameters: P          kappa       theta       lambda
                          10          0.5           1           1
Wage fn parameters:       gamma0    gamma1      gamma2      gamma3     gamma4
                          2       0.065     -0.0014        0.15      -0.004
Average value of private good - x:                     8.8998
Average value of wage - w:                             11.337
Average value of exogenous income - y:                 1
Average value of public good - G:                      2.8993
Average value of time in public good production (h):   0.29951
Estimated average individual utility:                  5.0776

***************************************************************************
Finished at 10-Jun-2005 00:08:42
Total estimation time 689.656 seconds
```

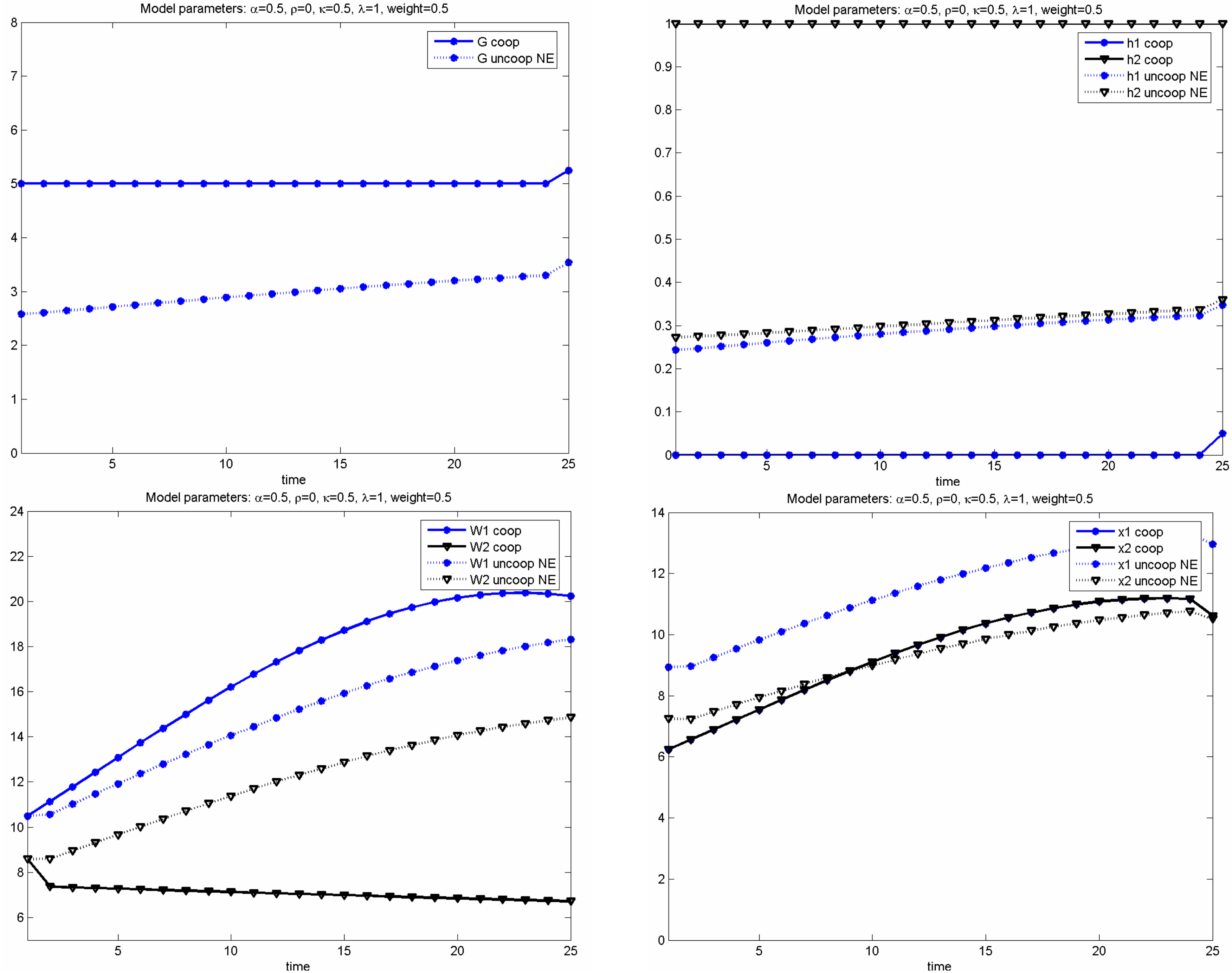
```
Started at 09-Jun-2005 23:44:37
***************************************************************************
********  Intrahousehold Allocation Model with Learning by Doing **********
********************   Cooperative solution   ****************************

Parameters for player     1
Time_periods             25
Population_size          80
Max_GA_generation      10000
Utility fn parameters:    betap      alphap      rho
                          0.95238         0.5           0
Production fn parameters: P          kappa       theta       lambda
                          10          0.5           1           1
Wage fn parameters:       gamma0    gamma1      gamma2      gamma3     gamma4
                          2.2       0.065     -0.0014        0.15      -0.004
Average value of private good - x:                     8.8874
Average value of wage - w:                             15.7957
Average value of exogenous income - y:                 1
Average value of public good - G:                      5.0052
Average value of time in public good production (h):   0.0010354
Estimated average individual utility:                  6.6401

Parameters for player 2
Utility fn parameters:    betap       alphap       rho
                          0.95238         0.5           0
Production fn parameters: P          kappa       theta       lambda
                          10          0.5           1           1
Wage fn parameters:       gamma0    gamma1      gamma2      gamma3     gamma4
                          2       0.065     -0.0014        0.15      -0.004
Average value of private good - x:                     8.8873
Average value of wage - w:                             7.1964
Average value of exogenous income - y:                 1
Average value of public good - G:                      5.0052
Average value of time in public good production (h):   1
Estimated average individual utility:                  6.6401
Estimated average joint utility:                       6.6401

***************************************************************************
Finished at 09-Jun-2005 23:56:00
Total estimation time 683.031 seconds..
```

Figure A1 (a) –(d)


Model parameters: α=0.5, ρ=0, κ=0.5, λ=1, weight=0.5
G coop
G uncoop NE
time
Model parameters: α=0.5, ρ=0, κ=0.5, λ=1, weight=0.5
h1 coop
h2 coop
h1 uncoop NE
h2 uncoop NE
time
Model parameters: α=0.5, ρ=0, κ=0.5, λ=1, weight=0.5
W1 coop
W2 coop
W1 uncoop NE
W2 uncoop NE
time
Model parameters: α=0.5, ρ=0, κ=0.5, λ=1, weight=0.5
x1 coop
x2 coop
x1 uncoop NE
x2 uncoop NE
time

Case 2

```
Started at 09-Jun-2005 13:20:16
***************************************************************************
********  Intrahousehold Allocation Model with Learning by Doing **********
**************************** Player 1 ************************************

Parameters for player    1
Time_periods            25
Population_size         80
Max_GA_generation     4000
Utility fn parameters:    beta       alpha      rho
                          0.95238         0.5         -5
Production fn parameters: P          kappa       theta       lambda
                          10          0.5           1          -10
Wage fn parameters:       gamma0    gamma1      gamma2      gamma3      gamma4
                          2.2       0.065     -0.0014        0.15      -0.004
Average value of private good - x:                      6.2943
Average value of wage - w:                              11.9784
Average value of exogenous income - y:                  1
Average value of public good - G:                       5.2335
Average value of time in public good production (h):   0.55525
Estimated average individual utility:                   5.6224

***************************************************************************
Finished at 09-Jun-2005 13:29:20
Total estimation time 543.203 seconds.

Started at 09-Jun-2005 13:20:19
***************************************************************************
********  Intrahousehold Allocation Model with Learning by Doing **********
**************************** Player 2 ************************************

Parameters for player    2
Time_periods            25
Population_size         80
Max_GA_generation     4000
Utility fn parameters:    beta      alpha      rho
                          0.95238         0.5         -5
Production fn parameters: P          kappa       theta       lambda
                          10          0.5           1          -10
Wage fn parameters:       gamma0    gamma1      gamma2      gamma3      gamma4
                          2       0.065     -0.0014        0.15      -0.004
Average value of private good - x:                      6.1476
Average value of wage - w:                              10.2109
Average value of exogenous income - y:                  1
Average value of public good - G:                       5.2335
Average value of time in public good production (h):   0.49144
Estimated average individual utility:                   5.5832

***************************************************************************
Finished at 09-Jun-2005 13:29:19
Total estimation time 540.531 seconds.
```

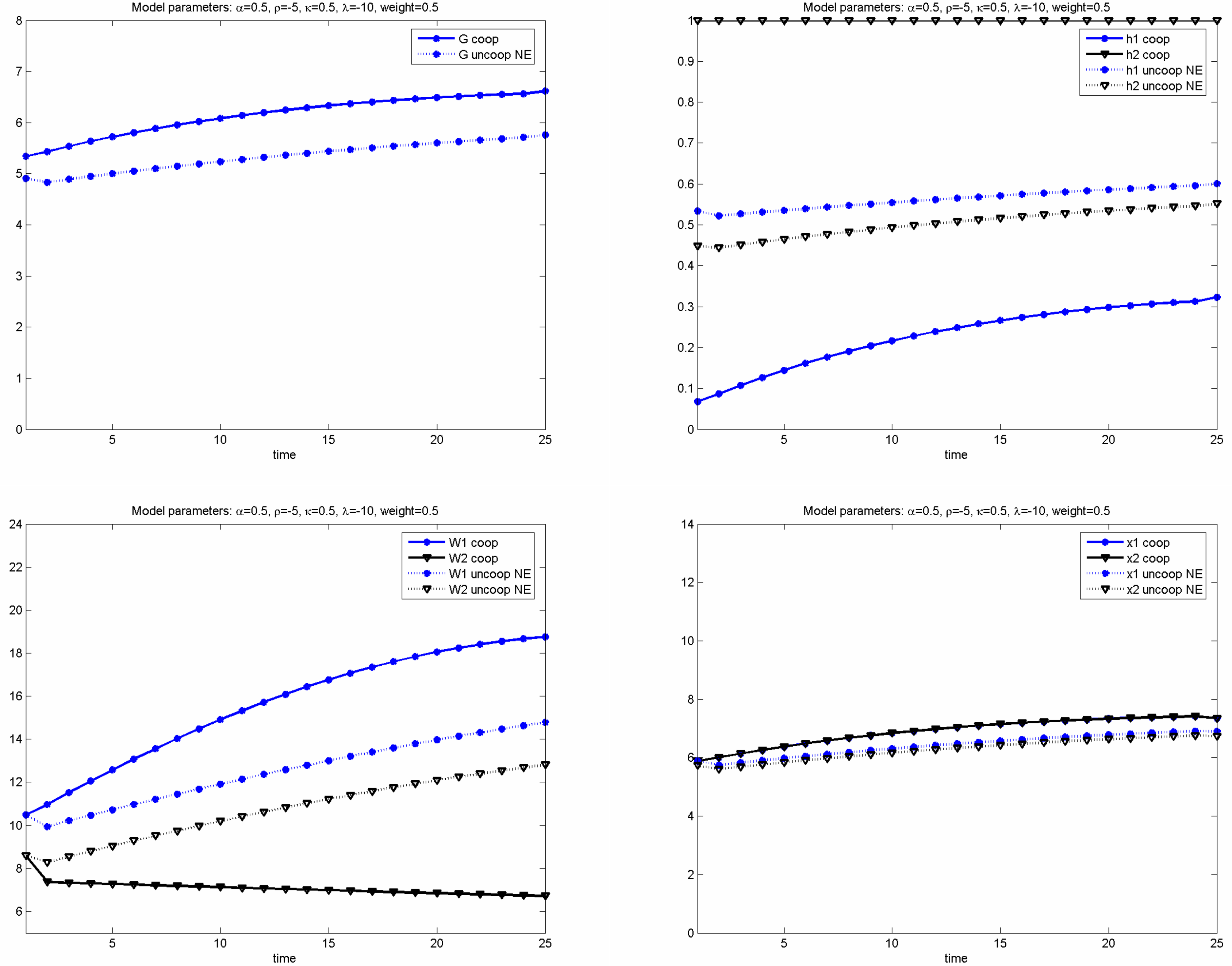
```
Started at 09-Jun-2005 13:01:47
***************************************************************************
********  Intrahousehold Allocation Model with Learning by Doing **********
********************   Cooperative solution   ****************************

Parameters for player     1
Time_periods             25
Population_size          80
Max_GA_generation     15000
Utility fn parameters:    betap      alphap      rho
                          0.95238         0.5         -5
Production fn parameters: P          kappa       theta       lambda
                          10          0.5           1          -10
Wage fn parameters:       gamma0    gamma1      gamma2      gamma3      gamma4
                          2.2       0.065     -0.0014        0.15      -0.004
Average value of private good - x:                      6.7379
Average value of wage - w:                              14.6456
Average value of exogenous income - y:                  1
Average value of public good - G:                       6.0123
Average value of time in public good production (h):   0.20246
Estimated average individual utility:                   6.3137

Parameters for player 2
Utility fn parameters:    betap       alphap       rho
                          0.95238         0.5         -5
Production fn parameters: P          kappa       theta       lambda
                          10          0.5           1          -10
Wage fn parameters:       gamma0    gamma1      gamma2      gamma3      gamma4
                          2       0.065     -0.0014        0.15      -0.004
Average value of private good - x:                      6.7377
Average value of wage - w:                              7.1964
Average value of exogenous income - y:                  1
Average value of public good - G:                       6.0123
Average value of time in public good production (h):   1
Estimated average individual utility:                   6.3137
Estimated average joint utility:                        6.3137

***************************************************************************
Finished at 09-Jun-2005 13:19:30
Total estimation time 1062.812 seconds.
```

Figure A2 (a) –(d)


Model parameters: α=0.5, ρ=-5, κ=0.5, λ=-10, weight=0.5
G coop
G uncoop NE
time
Model parameters: α=0.5, ρ=-5, κ=0.5, λ=-10, weight=0.5
h1 coop
h2 coop
h1 uncoop NE
h2 uncoop NE
time
Model parameters: α=0.5, ρ=-5, κ=0.5, λ=-10, weight=0.5
W1 coop
W2 coop
W1 uncoop NE
W2 uncoop NE
time
Model parameters: α=0.5, ρ=-5, κ=0.5, λ=-10, weight=0.5
x1 coop
x2 coop
x1 uncoop NE
x2 uncoop NE
time

Case 3

```
Started at 09-Jun-2005 13:44:51
***************************************************************************
********  Intrahousehold Allocation Model with Learning by Doing **********
***************************** Player 1 ************************************

Parameters for player    1
Time_periods            25
Population_size         80
Max_GA_generation      4000
Utility fn parameters:    beta       alpha      rho
                          0.95238         0.5          -5
Production fn parameters: P           kappa       theta       lambda
                          10           0.3            1          -10
Wage fn parameters:       gamma0    gamma1      gamma2      gamma3      gamma4
                          2.2       0.065     -0.0014        0.15      -0.004
Average value of private good - x:                     6.8574
Average value of wage - w:                             12.2622
Average value of exogenous income - y:                 1
Average value of public good - G:                      5.2071
Average value of time in public good production (h):   0.5186
Estimated average individual utility:                  5.718

***************************************************************************
Finished at 09-Jun-2005 13:54:46
Total estimation time 595.687 seconds.

Started at 09-Jun-2005 13:44:55
***************************************************************************
********  Intrahousehold Allocation Model with Learning by Doing **********
***************************** Player 2 ************************************

Parameters for player    2
Time_periods            25
Population_size         80
Max_GA_generation      4000
Utility fn parameters:    beta      alpha      rho
                          0.95238         0.5          -5
Production fn parameters: P           kappa       theta       lambda
                          10           0.3            1          -10
Wage fn parameters:      gamma0    gamma1      gamma2      gamma3      gamma4
                          2      0.065     -0.0014        0.15      -0.004
Average value of private good - x:                     5.7575
Average value of wage - w:                             10.0188
Average value of exogenous income - y:                 1
Average value of public good - G:                      5.2071
Average value of time in public good production (h):   0.52161
Estimated average individual utility:                  5.4412

***************************************************************************
Finished at 09-Jun-2005 13:54:46
Total estimation time 591 seconds.
```

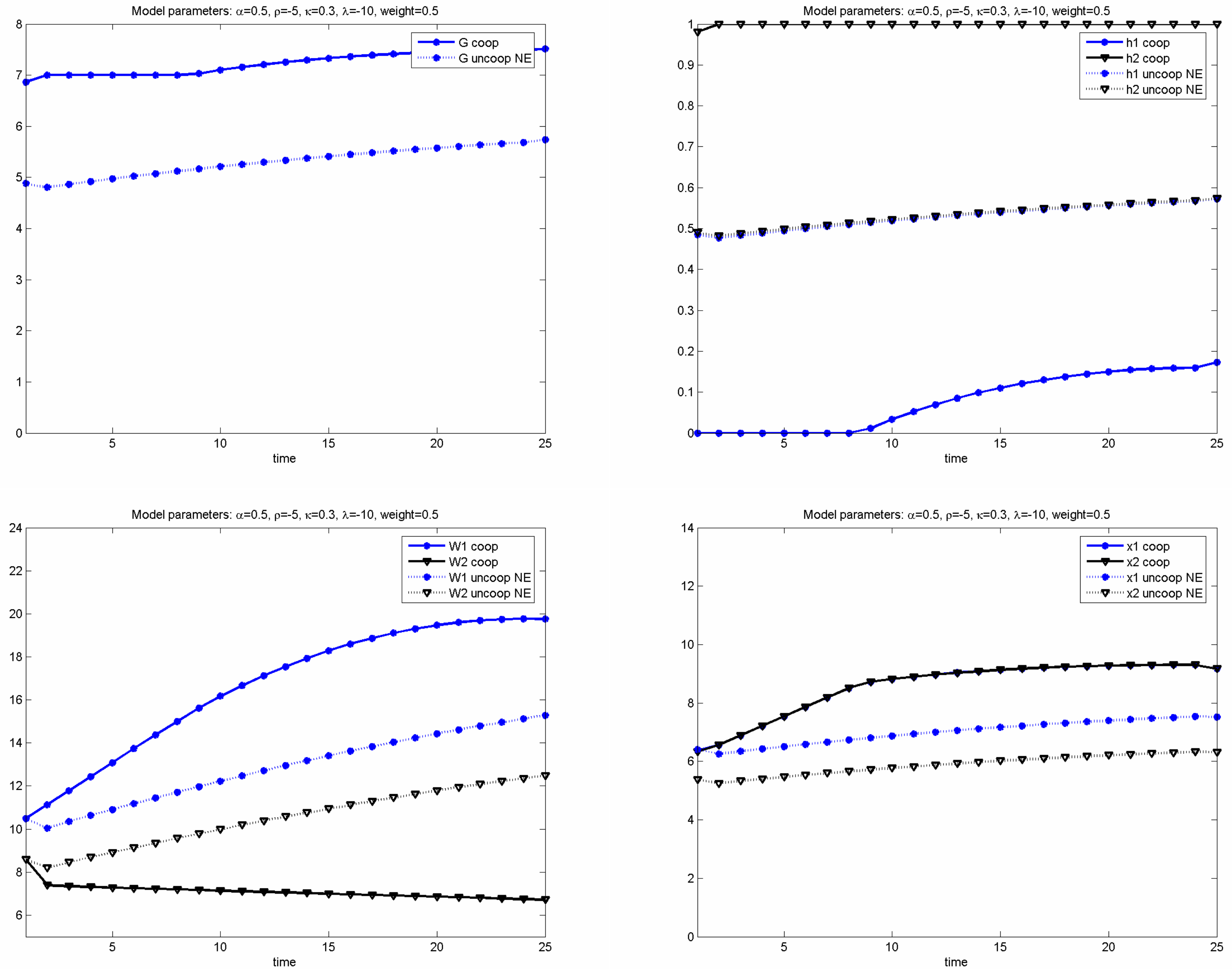
```
Started at 09-Jun-2005 13:30:36
***************************************************************************
********  Intrahousehold Allocation Model with Learning by Doing **********
********************    Cooperative solution    ****************************

Parameters for player     1
Time_periods            25
Population_size         80
Max_GA_generation     10000
Utility fn parameters:    betap      alphap      rho
                          0.95238         0.5          -5
Production fn parameters: P           kappa       theta       lambda
                          10           0.3            1          -10
Wage fn parameters:       gamma0    gamma1      gamma2      gamma3      gamma4
                          2.2       0.065     -0.0014        0.15      -0.004
Average value of private good - x:                     8.2744
Average value of wage - w:                             15.5819
Average value of exogenous income - y:                 1
Average value of public good - G:                      7.1585
Average value of time in public good production (h):   0.055917
Estimated average individual utility:                  7.5408

Parameters for player 2
Utility fn parameters:    betap       alphap       rho
                          0.95238         0.5          -5
Production fn parameters: P           kappa       theta       lambda
                          10           0.3            1          -10
Wage fn parameters:       gamma0    gamma1      gamma2      gamma3      gamma4
                          2      0.065     -0.0014        0.15      -0.004
Average value of private good - x:                     8.2715
Average value of wage - w:                             7.2062
Average value of exogenous income - y:                 1
Average value of public good - G:                      7.1585
Average value of time in public good production (h):   0.99868
Estimated average individual utility:                  7.5392
Estimated average joint utility:                       7.5392

***************************************************************************
Finished at 09-Jun-2005 13:43:13
Total estimation time 756.156 seconds.
```

Figure A3 (a) –(d)


Model parameters: α=0.5, ρ=-5, κ=0.3, λ=-10, weight=0.5
G coop
G uncoop NE
time
Model parameters: α=0.5, ρ=-5, κ=0.3, λ=-10, weight=0.5
h1 coop
h2 coop
h1 uncoop NE
h2 uncoop NE
time
Model parameters: α=0.5, ρ=-5, κ=0.3, λ=-10, weight=0.5
W1 coop
W2 coop
W1 uncoop NE
W2 uncoop NE
time
Model parameters: α=0.5, ρ=-5, κ=0.3, λ=-10, weight=0.5
x1 coop
x2 coop
x1 uncoop NE
x2 uncoop NE
time

Case 4

```
Started at 09-Jun-2005 14:21:32
***************************************************************************
********  Intrahousehold Allocation Model with Learning by Doing **********
**************************** Player 1 ***********************************

Parameters for player    1
Time_periods            25
Population_size         80
Max_GA_generation     4000
Utility fn parameters:    beta       alpha      rho
                          0.95238         0.5          -5
Production fn parameters: P          kappa       theta       lambda
                          10          0.3           1          -10
Wage fn parameters:       gamma0    gamma1      gamma2      gamma3     gamma4
                          2.2       0.065     -0.0014        0.15      -0.004
Average value of private good - x:                     7.2255
Average value of wage - w:                             12.9351
Average value of exogenous income - y:                 0
Average value of public good - G:                      5.4212
Average value of time in public good production (h):   0.43394
Estimated average individual utility:                  5.9671

***************************************************************************
Finished at 09-Jun-2005 14:30:51
Total estimation time 558.844 seconds.

Started at 09-Jun-2005 14:21:34
***************************************************************************
********  Intrahousehold Allocation Model with Learning by Doing **********
**************************** Player 2 ***********************************

Parameters for player    2
Time_periods            25
Population_size         80
Max_GA_generation     4000
Utility fn parameters:    beta      alpha      rho
                          0.95238         0.5          -5
Production fn parameters: P          kappa       theta       lambda
                          10          0.3           1          -10
Wage fn parameters:       gamma0    gamma1      gamma2      gamma3     gamma4
                          2      0.065     -0.0014        0.15      -0.004
Average value of private good - x:                     5.9312
Average value of wage - w:                             9.5915
Average value of exogenous income - y:                 2
Average value of public good - G:                      5.4212
Average value of time in public good production (h):   0.58849
Estimated average individual utility:                  5.642

***************************************************************************
Finished at 09-Jun-2005 14:30:51
Total estimation time 556.531 seconds.
```

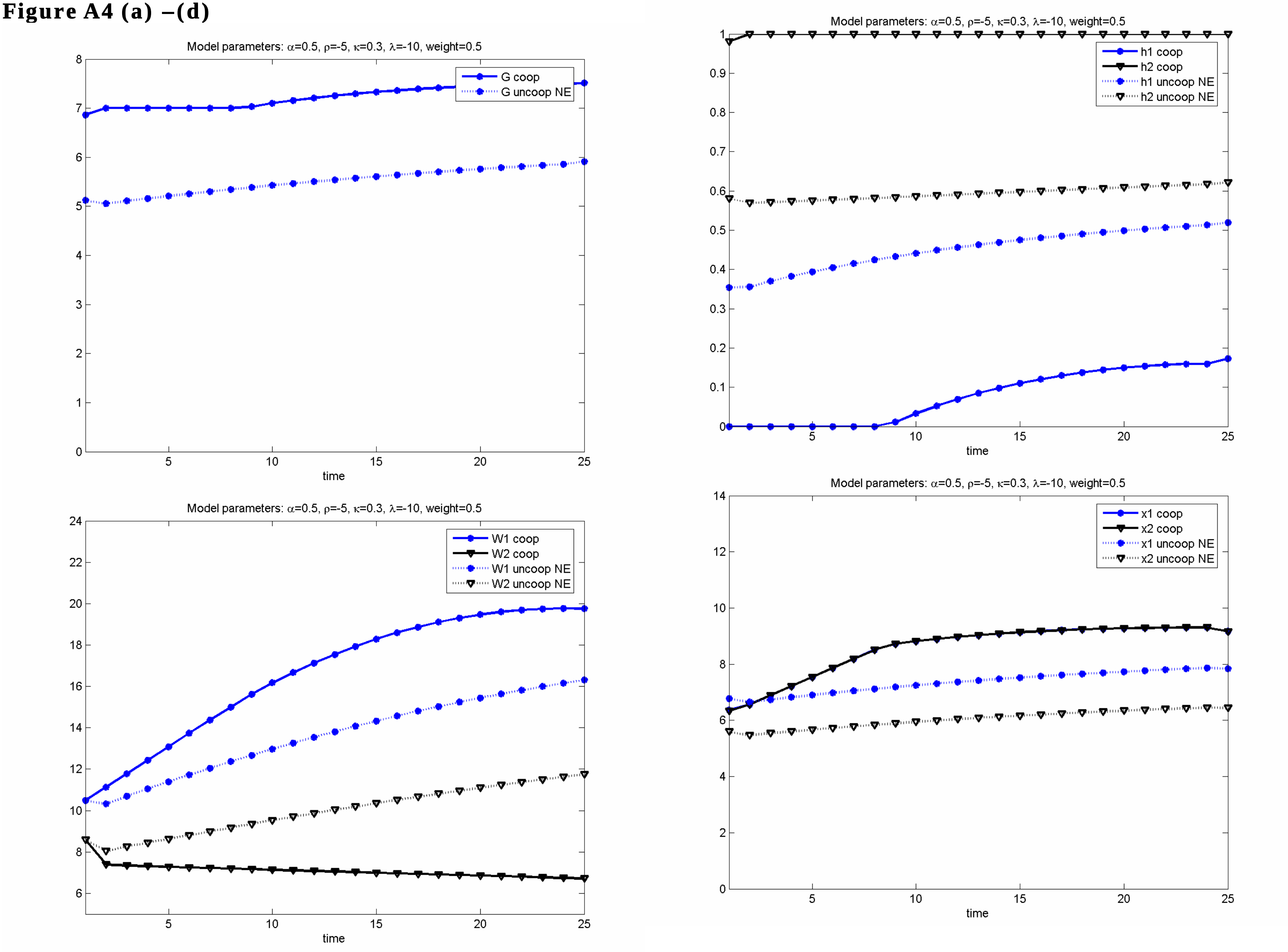
```
Started at 09-Jun-2005 14:01:16
***************************************************************************
********  Intrahousehold Allocation Model with Learning by Doing **********
********************   Cooperative solution   ****************************

Parameters for player     1
Time_periods            25
Population_size         80
Max_GA_generation     10000
Utility fn parameters:    betap      alphap      rho
                          0.95238         0.5          -5
Production fn parameters: P          kappa       theta       lambda
                          10          0.3           1          -10
Wage fn parameters:       gamma0    gamma1      gamma2      gamma3     gamma4
                          2.2       0.065     -0.0014        0.15      -0.004
Average value of private good - x:                     8.2748
Average value of wage - w:                             15.5821
Average value of exogenous income - y:                 0
Average value of public good - G:                      7.1584
Average value of time in public good production (h):   0.055884
Estimated average individual utility:                  7.5408

Parameters for player 2
Utility fn parameters:    betap       alphap       rho
                          0.95238         0.5          -5
Production fn parameters: P          kappa       theta       lambda
                          10          0.3           1          -10
Wage fn parameters:       gamma0    gamma1      gamma2      gamma3     gamma4
                          2      0.065     -0.0014        0.15      -0.004
Average value of private good - x:                     8.2719
Average value of wage - w:                             7.2062
Average value of exogenous income - y:                 2
Average value of public good - G:                      7.1584
Average value of time in public good production (h):   0.99868
Estimated average individual utility:                  7.5391
Estimated average joint utility:                       7.5392

***************************************************************************
Finished at 09-Jun-2005 14:14:26
Total estimation time 790.579 seconds.
```

Figure A4 (a) –(d)


Model parameters: α=0.5, ρ=-5, κ=0.3, λ=-10, weight=0.5
G coop
G uncoop NE
time
Model parameters: α=0.5, ρ=-5, κ=0.3, λ=-10, weight=0.5
h1 coop
h2 coop
h1 uncoop NE
h2 uncoop NE
time
Model parameters: α=0.5, ρ=-5, κ=0.3, λ=-10, weight=0.5
W1 coop
W2 coop
W1 uncoop NE
W2 uncoop NE
time
Model parameters: α=0.5, ρ=-5, κ=0.3, λ=-10, weight=0.5
x1 coop
x2 coop
x1 uncoop NE
x2 uncoop NE
time

Appendix B

Optimization using Genetic Algorithms[1]

Chambers (1995) describes Genetic Algorithm (GA) as an optimization method that uses genetics as its model of problem solving. Potential solution to the problem is treated as a genome (or chromosome), and GA creates a whole population of such solutions (chromosomes). The objective function, which we are maximizing, determines the fitness of chromosome (or how good each individual is). Evolution is controlled by natural selection: more fitted individuals are more likely to reproduce and give birth to more fitted offspring, while mutation helps to adapt to changing Nature conditions.

Using some selection criteria (usually biased towards better individuals, or individuals that are not very good but might have some good genetic material) GA picks individuals to mating pool. Then genetic operators such as mutation and crossover are applied to evolve the solutions in order to find the best individuals (the ones which maximize fitness - objective function). Crossover operator (usually) randomly selects two individuals (the parents) and combines their genes to produce two more individuals (the children). The purpose of crossover is to get genetic material from the previous generation to the subsequent generation. The mutation operator introduces a certain amount of randomness to the search. Mutation can create individual characteristics that may be advantageous and thus can help the search find solutions that crossover alone might not encounter. So, GA is a global stochastic search algorithm and it is less likely to get 'caught' at local optima than gradient search methods. Moreover, GA is perfect for corner solutions, as it does not require evaluating derivatives which might not exist at the corner points.

The scheme below, adapted from Alemdar and Ozyildirim (2000), describes the main steps involved in optimization with Genetic Algorithms. We use two parallel GA players to solve for non-cooperative Nash equilibrium, where each GA player maximizes hiss own utility treating choices of the other GA player as given. In order to achieve the

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1 Adapted from Wall, M. "Introduction to Genetic Algorithms", http://lancet.mit.edu/~mbwall/presentations/IntroToGAs/index.html

equilibrium both players have to exchange their best chromosomes (choices) simultaneously, so that fitness functions could be correctly evaluated:

1. initialize a (usually random) population of individuals
2. evaluate fitness of all initial individuals of population
3. (a) send the best own population member to shared memory
   (b) synchronization – wait until the other player has sent his best population member
   (b) receive the best population member from the other player from the shared memory
   (d) re-evaluate fitnesses of the population members to account for the new loaded chromosome of the other GA player
4. select a sub-population (mating pool) for offspring production
5. perform crossover: recombine the "genes" of selected parents
6. perform mutation: perturb the mated population stochastically
7. evaluate fitnesses of new individuals
8. select individuals who survive to the next generation
9. repeat steps (3) –(8) until some termination criterion satisfied

Steps 3 (a) to (d) are required only for non-cooperative solution. Cooperative solution is solved by a single GA player, who maximizes joint fitness function, so there is no need to exchange information with other GAs.

Synchronization is made by defining an indicator variable ‘$control_i$’ in shared memory i for each player ‘i’. Procedure works as follows: player 1 can send his best population member to shared memory 1 only if $control_1 = 0$ (i.e. player 2 has received the previous value); when he sends his best member, he sets $control_1 = 1$. Player 2 can read from shared memory 1 only after he sent his best player to shared memory 2. Then he can read from shared memory 1 only if $control_1 = 1$ (which means that player 1 sent a new member); when he receives player’s 1 best member, he sets $control_1 = 0$. Similarly, player 2 can sent his own best member to shared memory 2 only if $control_2 = 0$ (i.e. player 1 has received the previous value) and so on.

Appendix C

Matlab Code

parameters.m

```
function parameters
% This function creates parameters file 'parameters.mat', which is used for
% the Intrahousehold Allocation Game
% You should execute this file before executing any other files
% You should modify this file if you want to change game parameters
%
% created by 'Giedrius Blazys'
% at the 'University of Washington'
% for 2005 Spring Computational project

%% Define parameter values:

%% To solve the problem using GA:
M = 25;                % Number of time periods
N = 80;                % GA Population size
genMAX = 10000;        % Number of maximum GA generations

% Set exogenous income paths
y1=0*ones(1,M);
y2=0*ones(1,M);

%% CES Public Good production function parameters:
P=10;        % scale parameter
kappa=0.5;  % person's 1 relative productivity parameter
lambda=1;   % degree of substitutability (increases with lambda)
theta=1;    % returns to scale parameter
options_prod=[P kappa theta lambda];

weight=0.5;  %% relative Weight on husband’s utility function in cooperative case

%% Parameter values for spouse 1:

sdr=0.05;    % Subjective discount rate
betap=1/(1+sdr);
% CES Utility function parameters:
alphap=0.5;  % relative taste for private good parameter
rho=0;      % degree of substitutability (increases with rho)
options_util=[betap alphap rho];
% Wage determination function parameters (lnwage=gamma0+gamma1*EXPER+gamma2*EXPER^2+gamma3*
(1-htlagged)+gamma4*time)
gamma0=2.2; gamma1=0.065; gamma2=-0.0014; gamma3=0.15; gamma4=-0.004;
```

```
options_wage=[gamma0 gamma1 gamma2 gamma3 gamma4];

% Set parameter values for fitness function for spouse 1,
% options [beta alpha rho P kappa theta lambda gamma0 gamma1 gamma2 gamma3 gamma4]
evalOps1=[options_util options_prod options_wage];

%% Parameter values for spouse 2:

sdr=0.05;    % Subjective discount rate
betap=1/(1+sdr);
% CES Utility function parameters:
alphap=0.5;  % relative taste for private good parameter
rho=1;     % degree of substitutability (increases with rho)
options_util=[betap alphap rho];
% Wage determination function parameters (lnwage=gamma0+gamma1*EXPER+gamma2*EXPER^2+gamma3*
(1-htlagged)+gamma4*time)
gamma0=2.2; gamma1=0.065; gamma2=-0.0014; gamma3=0.15; gamma4=-0.004;
options_wage=[gamma0 gamma1 gamma2 gamma3 gamma4];

% Set parameter values for fitness function for spouse 2,
% options [beta alpha rho P kappa theta lambda gamma0 gamma1 gamma2 gamma3 gamma4]
evalOps2=[options_util options_prod options_wage];

%% Parameter values for GA optimization
% Crossover Operators
xFns = 'arithXover heuristicXover simpleXover';
xOpts = [1 0; 1 3; 1 0];
% Mutation Operators
mFns = 'boundaryMutation multiNonUnifMutation nonUnifMutation unifMutation swapMutation';
mOpts = [3 0 0;6 genMAX 2;6 genMAX 2;3 0 0;1 0 0];
% mOpts = [ceil(N*0.01) 0 0;1 genMAX/2 2;1 genMAX/5 2;1 0 0];
% Termination Operators
termFns = 'maxGenTerm';
termOps = [genMAX]; % genMAX Generations
% Selection Function
%selectFn = 'tournSelect';
%selectOps = [4]; %number of tournaments
selectFn = 'normGeomSelect';
selectOps = [0.02];
% GA Options [precision float/binary display]
gaOpts=[1e-15 1 0];
%% save parameters to parameters.mat
save parameters.mat M N genMAX y1 y2 weight evalOps1 evalOps2 ...
    xFns xOpts mFns mOpts termFns termOps selectFn selectOps gaOpts;
disp('Parameters file "parameters.mat" successfully created!');
clear;
```

sharedmemory.m

```
function [bestfrom1,bestfrom2]=sharedmemory(size,id)
% This function initializes shared memory of correct size (M+2)
% size - amount of allocated memory should be equal to M+2
% (control variable for synchronization, number of time periods,
% and the value of fitness function).
% id - player id
%
% created by 'Giedrius Blazys'
% at the 'University of Washington'
% for 2005 Spring Computational project

filename1=fullfile(pwd,'bestfrom1.dat');
filename2=fullfile(pwd,'bestfrom2.dat');

if id==1
    if ~exist(filename1,'file')
        [f1,msg]=fopen(filename1,'wb');
        if f1~=-1
            fwrite(f1,(zeros(1,size)),'double');
            fclose(f1);
        else
            error(['cannot open file ',filename1]);
        end
    end
    if ~exist(filename2,'file')
        [f2,msg]=fopen(filename2,'wb');
        if f2~=-1
            fwrite(f2,(zeros(1,size)),'double');
            fclose(f2);
        else
            error(['cannot open file ',filename2]);
        end
    end
    bestfrom1 = memmapfile(filename1,'format','double','writable',true);
    bestfrom2 = memmapfile(filename2,'format','double','writable',true);
elseif id==2
    if exist(filename1,'file')
        bestfrom1 = memmapfile(filename1,'format','double','writable',true);
    else, error('Start code with player 1!!');
    end
    if exist(filename2,'file')
        bestfrom2 = memmapfile(filename2,'format','double','writable',true);
    else, error('Start code with player 1!!');
    end
else, error('Incorrect player number!');
end
```

player1.m

```
% Non-cooperative Game
% Maximization for Player 1  (Husband)
%
% created by 'Giedrius Blazys'
% at the 'University of Washington'
% for 2005 Spring Computational project

global hspouse2 y1 bestfrom1 bestfrom2

parameters;
id=1;  %% Important! - set the correct player number
if id==1, spouseid=2;
elseif id==2, spouseid=1;
else, error('Incorrect player number!'); end

% Clears the shared memory files, so that new one of the correct size can be created
if id==1
    if exist('bestfrom1.dat','file')==2
        delete bestfrom1.dat;
    end
    if exist('bestfrom2.dat','file')==2
        delete bestfrom2.dat;
    end
end

t0 = clock;

if exist('output_pl1.log','file')==2, delete output_pl1.log; end
diary output_pl1.log

disp(['Started at ',datestr(now)]);
disp(['***************************************************************************']);
disp(['********  Intrahousehold Allocation Model with Learning by Doing **********']);
disp(['***************************** Player ',num2str(id),' ************************************']);
format short;

% Load parameter values
if exist('parameters.mat','file')==2
   load parameters.mat;
else, error('Parameters file "parameters.mat" not found!'); end

hsBounds=[ones(M,1)*[0 1]]; %Bounds for: time devoted to production of public good;

% Set parameter values for fitness function for person 1:
% options [beta alpha rho P kappa theta lambda gamma0 gamma1 gamma2 gamma3 gamma4]
evalOps=evalOps1;

[bestfrom1,bestfrom2]=sharedmemory(M+2,id); % initializes shared memory of correct size (M+2)

% Save a fictious best initial own population member
bestmember=0.5*ones(M,1);
bestfrom1.data=[1;bestmember;0];

% Load a fictious best initial population member from the other player
while ~bestfrom2.data(1)   % Synchronization - wait until the other GA player saves his best player
    pause(0.01);    % Pause before retry
end
hspouse2=bestfrom2.data(2:M+1);  % Spouce time contributions towards the public good
bestfrom2.data(1)=0;    % Allow the other GA player to save his best player

% startPop = initializega(N,hsBounds,'utility1',evalOps,[gaOpts(1) 1]);
% Creating starting population
startPop=(hsBounds(1,1)+(hsBounds(1,2)-hsBounds(1,1)).*rand(N,1))*ones(1,M+1); % Distributed uniformly
between bounds
[temp,startPop(:,end)]=utility1(startPop,[1 evalOps]);   % the last term is the value of fitness function

diary off;
%[hs,endPops,hsPop,traceInfo] = ga1(hsBounds,'utility1',evalOps,startPop,gaOpts,...
%                            termFns,termOps);  %GA maximixation
[hs,endPops,hsPop,traceInfo] = ga1(hsBounds,'utility1',evalOps,startPop,gaOpts,...
```

```
                              termFns,termOps,selectFn,selectOps,xFns,xOpts,mFns,mOpts);  %GA maximixation
diary on;

%% results
betap=evalOps(1); alphap=evalOps(2); rho=evalOps(3);
A=evalOps(8); b=evalOps(9); c=evalOps(10); d=evalOps(11); e=evalOps(12); % shape parameters for wage
function
% shape parameters for G production function
options_prod=evalOps(4:7);

t=1:M;
hlag=[0,hs(1:(M-1))];
L=[0,(1-hs(1:M))*triu(ones(M,M))];  % Cummulated work experience
w=exp(A+b*L(1:(end-1))+c*(L(1:(end-1)).^2)+d*(1-hlag)+e*(t-1)); %Productivity increases with accumulated
work experience
x=w.*(1-hs(1:M))+y1;       %Private good
G=produce(hs(1:M),hspouse2',options_prod);  %Production of Public good requires time input of both spouces
h=[hs(1:M)];
betam=ones(1,M);
for i=2:M, betam(i)=betam(i-1)*betap; end

disp(' ');
GApars={'Parameters for player ','Time_periods ','Population_size ','Max_GA_genration '};
disp([char(GApars'),num2str([id,M,N,genMAX]')]);
disp(['Utility fn parameters:    beta       alpha      rho']);
disp(['                          ',num2str(evalOps(1:3))]);
disp(['Production fn parameters: P          kappa       theta       lambda']);
disp(['                          ',num2str(evalOps(4:7))]);
disp(['Wage fn parameters:       A            b            c           d         e']);
disp(['                          ',num2str([A b c d e])]);
disp(['Average value of private good - x:                       ',num2str(sum(betam.*x)/sum(betam))]);
disp(['Average value of wage - w:                               ',num2str(sum(betam.*w)/sum(betam))]);
disp(['Average value of exogenous income - y:                   ',num2str(sum(betam.*y1)/sum(betam))]);
disp(['Average value of public good - G:                        ',num2str(sum(betam.*G)/sum(betam))]);
disp(['Average value of time in public good production (h):    ',num2str(sum(betam.*h)/sum(betam))]);
disp(['Estimated average individual utility:                    ',num2str(hs(end)/sum(betam))]);
disp(' ');

save results1uc.mat id betap alphap rho options_prod A b c d e h hspouse2 G x w;

disp(['**************************************************************************']);
disp(['Finished at ',datestr(now)]);
disp(['Total estimation time ',num2str(etime(clock,t0)),' seconds.']);
diary off;
clear global;
pause(2)
% Clears the shared memory files
if exist('bestfrom1.dat','file')==2
    delete bestfrom1.dat;
end
if exist('bestfrom2.dat','file')==2
    delete bestfrom2.dat;
end
disp('Finished!');
```

utility1.m

```
function [hsopt,val]=utility1(hsopt,options)
% Utility function for player1 (Husband)
%
% created by 'Giedrius Blazys'
% at the 'University of Washington'
% for 2005 Spring Computational project
%
% CES utility function of form: U(x,G)=(alpha*x^rho+(1-alpha)*G^rho)^(1/rho)
% rho<=1, the bigger the sigma, the greater the degree of substitutability
% between the commodities; alpha measures the relative preferece of x versus G.
% Each cromosome of hsopt is a row vector, where the last term is the
% fitness value, this is a vectorized function, so it works on the whole
% population of size N.
%
```

```
global hspouse2 y1
curr_gen=options(1);
betap=options(2);
alphap=options(3);
rho=options(4);
options_prod=options(5:8);
A=options(9); b=options(10); c=options(11); d=options(12); e=options(13);   % shape parameters for wage
function

M=(size(hsopt,2)-1); % We have 1 choice variable over M periods, while last column is for the value of
fitness funtion
N=(size(hsopt,1));   % This is a vectorized function, so it works on the whole population of size N
betam=ones(1,M);
for i=2:M, betam(i)=betam(i-1)*betap; end
betam=ones(N,1)*betam;
t=1:M;
t=ones(N,1)*t;
hlag=[zeros(N,1),hsopt(:,1:(M-1))];
L=[zeros(N,1),(1-hsopt(:,1:M))*triu(ones(M,M))];  % Cummulated work experience
w=exp(A+b.*L(:,1:(end-1))+c.*(L(:,1:(end-1)).^2)+d.*(1-hlag)+e.*(t-1)); %Productivity increases with
accumulated work experience
x=w.*(1-hsopt(:,1:M))+ones(N,1)*y1;       %Private good
G=produce(hsopt(:,1:M),(ones(N,1)*hspouse2'),options_prod);  %Production of Public good requires time
input of both spouses

if rho>1, error('rho has to be <=1');
elseif rho==0           %utility function becomes Cobb-Douglas
    Um=betam.*((x.^alphap).*(G.^(1-alphap)));
elseif rho<=-50        %utility function becomes Rawlsian
    Um=betam.*min(x,G);
else
    Um=betam.*((alphap.*(x.^rho)+(1-alphap).*(G.^rho)).^(1./rho));
end
val=sum(Um,2);
hsopt(:,end)=val;
```

produce.m

```
function G=produce(h1,h2,options)
% Production function for household public good
%
% created by 'Giedrius Blazys'
% at the 'University of Washington'
% for 2005 Spring Computational project
%
% options [P kappa theta lambda]
% CES production function: G=P*(kappa*(h1^lambda)+(1-kappa)*(h2^lambda))^(theta/lambda)
% this is a vectorized version of production function: h1 is NxM and h2 is
% NxM matrix.
%
P=options(1);           % scale parameter
kappa=options(2);       % person's 1 relative productivity parameter
lambda=options(3);      % degree of substitutability (increases with lambda)
theta=options(4);       % returns to scale parameter

% CES production function
if lambda>1, error(['lambda has to be <=1. You supplied: ',num2str(rho)]);
elseif lambda==0        %production function becomes Cobb-Douglas
    G=P.*((h1.^kappa).*(h2.^(1-kappa)));
elseif lambda<=-50     %production function becomes Rawlsian
    G=P.*min(h1,h2);
else
    G=P.*((kappa.*(h1.^lambda)+(1-kappa).*(h2.^lambda)).^(1./lambda));
end
```

ga1.m

```
function [x,endPop,bPop,traceInfo] = ga1(bounds,evalFN,evalOps,startPop,opts,...
termFN,termOps,selectFN,selectOps,xOverFNs,xOverOps,mutFNs,mutOps)
% GA for non-cooperative Game for Player 1  (Husband)
%
```

```
% modified by 'Giedrius Blazys'
% at the 'University of Washington'
% for 2005 Spring Computational project
%
global hspouse2 bestfrom1 bestfrom2
. . .
  if mod(gen,round(termOps/20))==0, disp(['Currently running generation ',num2str(gen),...
             ', ',num2str(termOps-gen),' generations left to run!']); end
. . .
    % Save own best population member
    while bestfrom1.data(1)   % Synchronization - wait until the other GA player read the
previous data
        pause(0.00001);    % Pause before retry
    end
    bestfrom1.data=[1;best'];     % Save the best own population member
    % Load the best population member from the other player
    while ~bestfrom2.data(1)   % Synchronization - wait until the other GA player saved his best
player
        pause(0.00001);    % Pause before retry
    end
    hspouse2=bestfrom2.data(2:numVar+1);  % Spouce time contributions towards the public good
    bestfrom2.data(1)=0;    % Allow the other GA player to save his best player

    %% Re-evaluate fitnesses of the population members to account for the
    %% new best cromosome of the other GA player
    eval(['[temp,startPop(:,end)]=' evalFN '(startPop,[1 evalOps]);']);
    [bval,bindx] = max(startPop(:,xZomeLength));
    best =  startPop(bindx,:);
. . .
```

player2.m

{similar as player1.m}

utility2.m

{similar as utility1.m}

ga2.m

{similar as ga1.m}

players.m

```
% Cooperative Game
% Maximization for both Players simultaneously (Husband and wife)
%
% created by 'Giedrius Blazys'
% at the 'University of Washington'
% for 2005 Spring Computational project

parameters;

global y y_s

t0 = clock;

if exist('output_both_pl.log','file')>0, delete output_both_pl.log; end
diary output_both_pl.log

disp(['Started at ',datestr(now)]);
disp(['******************************************************************************']);
disp(['********  Intrahousehold Allocation Model with Learning by Doing **********']);
disp(['********************   Cooperative solution   ****************************']);
disp(' ');
format short;
```

```
% Load parameter values

if exist('parameters.mat','file')==2
   load parameters.mat;
else, disp('Parameters file "parameters.mat" not found!');
end

hsBounds=[ones(2*M,1)*[0 1]; ones(M,1)*[0 100]]; %Bounds for h1t, h2t and x2t;

% Set parameter values for fitness function for both players:
% options [options_util options_prod options_wage weight options_util_s options_wage_s]
evalOps=[evalOps1 weight evalOps2(1:3) evalOps2(8:end)];

y=y1;       % Set exogeneous income path
y_s=y2;       % Set exogeneous income path

% startPop = initializega(N,hsBounds,'utility',evalOps,[gaOpts(1) 1]);
% Creating starting population
startPop=[(hsBounds(1,1)+(hsBounds(1,2)-hsBounds(1,1)).*rand(N,1))*ones(1,2*M) ...
    (hsBounds(2*M+1,1)+(hsBounds(2*M+1,2)-hsBounds(2*M+1,1)).*rand(N,1))*ones(1,M+1)]; % Distributed
uniformly between bounds
for i=1:N, [temp,startPop(i,end)]=utility(startPop(i,:),[1 evalOps]); end  % the last term is the value
of fitness function

diary off;
[hs,endPops,hsPop,traceInfo] = ga(hsBounds,'utility',evalOps,startPop,gaOpts,...
                            termFns,termOps,selectFn,selectOps,xFns,xOpts,mFns,mOpts);  %GA maximixation
diary on;

%% results for spouce 1:
id=1;
betap=evalOps(1); alphap=evalOps(2); rho=evalOps(3);
A=evalOps(8); b=evalOps(9); c=evalOps(10); d=evalOps(11); e=evalOps(12); % shape parameters for wage
function
options_prod=evalOps(4:7);      % shape parameters for G production function
hspouce=hs(M+1:2*M);
t=1:M;
hlag=[0,hs(1:(M-1))];
hlag_s=[0,hs(M+1:(2*M-1))];
L=[0,(1-hs(1:M))*triu(ones(M,M))];  % Cummulated work experience
L_s=[0,(1-hs(M+1:2*M))*triu(ones(M,M))];  % Cummulated work experience
w_s=exp(A+b*L_s(1:(end-1))+c*(L_s(1:(end-1)).^2)+d*(1-hlag_s)+e*(t-1)); %Productivity increases with
accumulated work experience

w=exp(A+b*L(1:(end-1))+c*(L(1:(end-1)).^2)+d*(1-hlag)+e*(t-1)); %Productivity increases with accumulated
work experience
x=w_s.*(1-hs(M+1:2*M))+y_s+w.*(1-hs(1:M))+y-hs(2*M+1:3*M);   %Private good
G=produce(hs(1:M),hspouce,options_prod);  %Production of Public good requires time input of both spouces
h=[hs(1:M)];

betam=ones(1,M);
for i=2:M, betam(i)=betam(i-1)*betap; end
if rho>1, error('rho has to be <=1');
elseif rho==0           %utility function becomes Cobb-Douglas
    Um=betam.*((x.^alphap).*(G.^(1-alphap)));
elseif rho<=-100        %utility function becomes Rawlsian
    Um=betam.*min(x,G);
else
    Um=betam.*((alphap.*(x.^rho)+(1-alphap).*(G.^rho)).^(1./rho));
end
Um=sum(Um);

disp(' ');
GApars={'Parameters for player ','Time_periods ','Population_size ','Max_GA_genration '};
disp([char(GApars'),num2str([id,M,N,genMAX]')]);
disp(['Utility fn parameters:    betap      alphap      rho']);
disp(['                          ',num2str(evalOps(1:3))]);
disp(['Production fn parameters: P          kappa       theta       lambda']);
disp(['                          ',num2str(evalOps(4:7))]);
disp(['Wage fn parameters:       A            b            c           d         e']);
disp(['                          ',num2str([A b c d e])]);
disp(['Average value of private good - x:                     ',num2str(sum(betam.*x)/sum(betam))]);
disp(['Average value of wage - w:                             ',num2str(sum(betam.*w)/sum(betam))]);
```

```
disp(['Average value of exogenous income - y:                 ',num2str(sum(betam.*y1)/sum(betam))]);
disp(['Average value of public good - G:                      ',num2str(sum(betam.*G)/sum(betam))]);
disp(['Average value of time in public good production (h):   ',num2str(sum(betam.*h)/sum(betam))]);
disp(['Estimated average individual utility:                  ',num2str(Um/sum(betam))]);
disp(' ');
save results1c.mat id betap alphap rho options_prod A b c d e h hspouce G x w weight;

%% results for spouce 2:
id=2;
betap=evalOps(14); alphap=evalOps(15); rho=evalOps(16);
A=evalOps(17); b=evalOps(18); c=evalOps(19); d=evalOps(20); e=evalOps(21); % shape parameters for wage
function
options_prod=evalOps(4:7);      % shape parameters for G production function

hspouce=hs(1:M);
t=1:M;
hlag=[0,hs(1:(M-1))];
hlag_s=[0,hs(M+1:(2*M-1))];
L=[0,(1-hs(1:M))*triu(ones(M,M))];  % Cummulated work experience
L_s=[0,(1-hs(M+1:2*M))*triu(ones(M,M))];  % Cummulated work experience
w=exp(A+b*L(1:(end-1))+c*(L(1:(end-1)).^2)+d*(1-hlag)+e*(t-1)); %Productivity increases with accumulated
work experience
w_s=exp(A+b*L_s(1:(end-1))+c*(L_s(1:(end-1)).^2)+d*(1-hlag_s)+e*(t-1)); %Productivity increases with
accumulated work experience

G=produce(hspouce,hs(M+1:2*M),options_prod);  %Production of Public good requires time input of both
spouces
x=hs(2*M+1:3*M);                          %Private good for spouse
h=[hs(M+1:2*M)];
w=w_s;

betam=ones(1,M);
for i=2:M, betam(i)=betam(i-1)*betap; end
if rho>1, error('rho has to be <=1');
elseif rho==0           %utility function becomes Cobb-Douglas
    Um=betam.*((x.^alphap).*(G.^(1-alphap)));
elseif rho<=-100        %utility function becomes Rawlsian
    Um=betam.*min(x,G);
else
    Um=betam.*((alphap.*(x.^rho)+(1-alphap).*(G.^rho)).^(1./rho));
end
Um=sum(Um);

disp(['Parameters for player ',num2str(id)]);
disp(['Utility fn parameters:    betap       alphap       rho']);
disp(['                          ',num2str(evalOps(14:16))]);
disp(['Production fn parameters: P          kappa       theta       lambda']);
disp(['                          ',num2str(evalOps(4:7))]);
disp(['Wage fn parameters:       A           b           c          d         e']);
disp(['                          ',num2str([A b c d e])]);
disp(['Average value of private good - x:                     ',num2str(sum(betam.*x)/sum(betam))]);
disp(['Average value of wage - w:                             ',num2str(sum(betam.*w)/sum(betam))]);
disp(['Average value of exogenous income - y:                 ',num2str(sum(betam.*y2)/sum(betam))]);
disp(['Average value of public good - G:                      ',num2str(sum(betam.*G)/sum(betam))]);
disp(['Average value of time in public good production (h):   ',num2str(sum(betam.*h)/sum(betam))]);
disp(['Estimated average individual utility:                  ',num2str(Um/sum(betam))]);
disp(['Estimated average joint utility:                       ',num2str(hs(end)/sum(betam))]);

disp(' ');
save results2c.mat id betap alphap rho options_prod A b c d e h hspouce G x w weight;

disp(['***************************************************************************']);
disp(['Finished at ',datestr(now)]);
disp(['Total estimation time ',num2str(etime(clock,t0)),' seconds.']);
diary off;
```

utility.m

```
function [hsopt,val]=utility(hsopt,options)
% Utility function for both players (Husband and Wife)
%
% created by 'Giedrius Blazys'
```

```
% at the 'University of Washington'
% for 2005 Spring Computational project
%
% CES utility function of form: U(x,G)=(alpha*x^rho+(1-alpha)*G^rho)^(1/rho)
% rho<=1, the bigger the sigma, the greater the degree of substitutability
% between the commodities; alpha measures the relative preferece of x versus G
%
global y y_s
curr_gen=options(1);
betap=options(2);
alphap=options(3);
rho=options(4);
options_prod=options(5:8);
A=options(9); b=options(10); c=options(11); d=options(12); e=options(13);   % shape parameters
for wage function
weight=options(14);
%% second person - wife (spouse):
betap_s=options(15);
alphap_s=options(16);
rho_s=options(17);
A_s=options(18); b_s=options(19); c_s=options(20); d_s=options(21); e_s=options(22); % shape
parameters for wage function

M=(length(hsopt)-1)/3; %We have 3 choice variable over M periods, while last column is for the
value of fitness funtion
betam=ones(1,M);
for i=2:M, betam(i)=betam(i-1)*betap; end
betam_s=ones(1,M);
for i=2:M, betam_s(i)=betam_s(i-1)*betap_s; end
L=[0,(1-hsopt(1:M))*triu(ones(M,M))];          % Cummulated work experience
L_s=[0,(1-hsopt(M+1:2*M))*triu(ones(M,M))];     % Cummulated work experience for spouse
t=1:M;
hlag=[0,hsopt(1:(M-1))];
hlag_s=[0,hsopt(M+1:(2*M-1))];
w=exp(A+b*L(1:(end-1))+c*(L(1:(end-1)).^2)+d*(1-hlag)+e*(t-1)); %Productivity increases with
accumulated work experience
w_s=exp(A_s+b_s*L_s(1:(end-1))+c_s*(L_s(1:(end-1)).^2)+d_s*(1-hlag_s)+e_s*(t-1)); %Productivity
increases with accumulated work experience
x_s=hsopt(2*M+1:3*M);                                  %Private good for spouse
x=w_s.*(1-hsopt(M+1:2*M))+y_s+w.*(1-hsopt(1:M))+y-x_s;   %Private good
%  replace non-positive x with small x=1e-5;
xold=x;
x=(sign(x)==1).*x+(1e-10)*(sign(x)~=1);
%  make sure that h1 and h2 are within bounds [0,1];
hsopt(1:2*M)=((hsopt(1:2*M))<=1).*hsopt(1:2*M)+(hsopt(1:2*M)>1);
G=produce(hsopt(1:M),hsopt(M+1:2*M),options_prod);  %Production of Public good requires time
input of both spouces
if rho>1, error('rho has to be <=1');
elseif rho==0          %utility function becomes Cobb-Douglas
    Um=betam.*((x.^alphap).*(G.^(1-alphap)));
elseif rho<=-100       %utility function becomes Rawlsian
    Um=betam.*min(x,G);
else
    Um=betam.*((alphap.*(x.^rho)+(1-alphap).*(G.^rho)).^(1./rho));
end
if rho_s>1, error('rho has to be <=1');
elseif rho_s==0        %utility function becomes Cobb-Douglas
    Um_s=betam_s.*((x_s.^alphap_s).*(G.^(1-alphap_s)));
elseif rho<=-100       %utility function becomes Rawlsian
    Um_s=betam_s.*min(x_s,G);
else
    Um_s=betam_s.*((alphap_s.*(x_s.^rho_s)+(1-alphap_s).*(G.^rho_s)).^(1./rho_s));
end
Um=Um-(1e+15)*((sign(xold)==-1).*xold.*xold); % Punish for negative x values
%% (sign(x)==-1) is a matrix of indicators of negative x values
val=sum([weight.*Um+(1-weight).*Um_s],2);        % weighted utility of both spouses
```